Influence of long-range correlated defects on critical ultrasound propagation in solids

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(Received 11 April 2009; revised manuscript received 9 June 2009; published 23 July 2009)

The effect of long-range correlated quenched structural defects on the critical ultrasound attenuation and sound velocity dispersion is studied for three-dimensional Ising-like systems. A field-theoretical description of the dynamic critical effects of ultrasound propagation in solids is performed with allowance for both fluctuation and relaxation attenuation mechanisms. The temperature and frequency dependences of the dynamical scaling functions of the ultrasound critical characteristics are calculated in a two-loop approximation for different values of the correlation parameter a of the Weinrib-Halperin model with long-range correlated defects. The asymptotic behavior of the dynamical scaling functions in hydrodynamic and critical regions is separated. The influence of long-range correlated disorder on the asymptotic behavior of the critical ultrasonic anomalies is discussed.

I. INTRODUCTION

The unique feature of ultrasonic methods consists in the fact that for a solid both an anomalous peak of ultrasonic attenuation and an anomalous change in the ultrasound velocity are observable from experiments[1,2] in vicinity of critical point. These phenomena are caused by the interaction of low-frequency acoustic oscillations with long-lived order parameter fluctuations, which produce a random force that perturbs normal acoustic modes by means of magnetostrictive spin-phonon interaction. In this process, relaxation and fluctuation attenuation mechanisms can be distinguished. The relaxation mechanism, which is due to a linear dynamic relationship between sound waves and an order parameter,[3] manifests itself only in an ordered phase, where the statistical average of the order parameter is nonzero. Since the relaxation of the order parameter near a phase-transition point proceeds slowly, this mechanism plays an important role in the dissipation of low-frequency acoustic oscillations. The fluctuation attenuation mechanism, which is determined by a quadratic relation between the deformation variables in the Hamiltonian of a system with order parameter fluctuations, manifests itself over the entire critical temperature range. To date, there are a considerable number of works that deal with a theoretical description of the ultrasonic critical anomalies,[4–6] which give an adequate explanation of experimental results.[7–9]

One of the most interesting and important problems from both experimental and theoretical viewpoints is the study of the influence of structural defects on the dynamical process of sound propagation in solids undergoing phase transitions. The structural disorder induced by impurities or other structural defects plays a key role in the behavior of real materials and physical systems. The structural disorder breaks the translational symmetry of the crystal and thus greatly complicates the theoretical description of the material. The influence of disorder is particularly important near critical point where behavior of a system is characterized by anomalous large response on any even weak perturbation. In most investigations consideration has been restricted to the case of point-like uncorrelated defects.[10] However, the nonidealities of structure cannot be modeled by simple uncorrelated defects only. Solids often contain defects of a more complex structure: linear dislocations, planar grain boundaries, clusters of point-like defects, and so on.

According to the Harris criterion,[11] the critical behavior of only Ising systems is changed by the presence of quenched point-like defects. The problem of the influence of point-like defects on the critical sound propagation in Ising systems has been discussed in paper[12] with the use of a ε expansion in the lowest order of approximation. However, our pilot analysis of this phenomenon showed that in Ref. 12 some diagrams which are needed for a correct description of the influence of the disorder were not considered. Furthermore, our numerous investigations of pure and disordered systems performed in the two-loop and higher orders of the approximation for the three-dimensional (3D) system directly, together with the use of methods of series summation, show that the predictions made in the lowest order of the approximation, especially on the basis of the ε expansion, can differ strongly from the real critical behavior.[13–18] In paper,[19] we have realized a correct field-theoretical description of dynamical effects of the influence of point-like defects on acoustic anomalies near temperature of the second-order phase transition for 3D Ising systems in the two-loop approximation with consideration of only the fluctuation attenuation mechanism. In papers[20,21] we have performed, the field-theoretical description of the influence of point-like defects on critical acoustic anomalies with allowance for both fluctuation and relaxation attenuation mechanisms. Some misprints, which were committed in the paper[20] in Eq. (20) for the scaling function of critical ultrasound attenuation, were corrected in the paper.[21] The temperature and frequency dependencies of the scaling functions for ultrasonic attenuation and sound velocity dispersion were calculated in paper[21] in the two-loop approximation with the use of the resummation of asymptotic series technique for pure and disordered systems, and their asymptotic behavior in hydrodynamic and critical regions were distinguished. It was shown that the presence of structural defects causes a stronger temperature and frequency dependencies of the acoustic characteristics in the critical region and increase in the critical anomalous peak of ultrasonic attenuation in comparison with pure system.

However, the question about influence of long-range (LR) correlation effects realizing in defects with complex structure on critical behavior of systems was investigated noticeably weaker, and this question has not been studied entirely in respect to display of correlation effects in the critical anomalous properties of sound propagation in solids. Different models of structural disorder have arisen as an attempt to describe complicated defects. In this paper, we concentrate on the model of Weinrib and Halperin (WH)[22] with the so-called LR-correlated disorder when pair correlation function for point-like defects $g(x-y)$ falls off with distance as a power law $g(x-y) \sim |x-y|^{-a}$. Weinrib and Halperin showed that for $a \geq d$ LR correlations are irrelevant and the usual short-range (SR) Harris criterion[11] $2-d\nu_0 = \alpha_0 > 0$ of the effect of point-like uncorrelated defects is realized, where d is the spatial dimension, and ν_0 and α_0 are the correlation-length and the specific-heat exponents of the pure system. For $a < d$ the extended criterion $2-a\nu_0 > 0$ of the effect of disorder on the critical behavior was established. As a result, a wider class of disordered systems, not only the 3D diluted Ising model with point-like uncorrelated defects, can be characterized by a type of critical behavior. So, for $a < d$ a LR-disorder stable fixed point (FP) of the renormalization group (RG) recursion relations for systems with a number of components of the order parameter $m \geq 2$ was discovered. The critical exponents were calculated in the one-loop approximation using a double expansion in $\varepsilon = 4-d \ll 1$ and $\delta = 4-a \ll 1$. The correlation-length exponent was evaluated in this linear approximation as $\nu = 2/a$ and it was argued that this scaling relation is exact and also holds in higher order approximation. In the case $m=1$ the accidental degeneracy of the recursion relations in the one-loop approximation did not permit to find LR-disorder stable FP. Korzhenevskii *et al.*[23] proved the existence of the LR-disorder stable FP for the one-component WH model and also found characteristics of this type of critical behavior.

The results for WH model with LR correlated defects received based on the ε, δ expansion[22–26] were questioned in our paper,[16] where a renormalization analysis of scaling functions was carried out directly for the 3D systems in the two-loop approximation with the values of a in the range $2 \leq a \leq 3$, and the FP's corresponding to stability of various types of critical behavior were identified. The static and dynamic critical exponents in the two-loop approximation were calculated with the use of the Padé-Borel summation technique. The results obtained in Ref. 16 essentially differ from the results evaluated by a double ε, δ expansion. The comparison of calculated the exponent ν values and ratio $2/a$ showed the violation of the relation $\nu = 2/a$, assumed in Ref. 22 as exact. Monte-Carlo simulations of the short-time dynamic behavior were carried out in paper[27] for 3D Ising and *XY* models with LR correlated disorder at criticality, in the case corresponding to linear defects with $a=2$. It was shown that the obtained values of the static and dynamic critical exponents are in a good agreement with our results of the field-theoretic description of the critical behavior of these models in Ref. 16.

The models with LR-correlated quenched defects have both theoretical interest due to the possibility of predicting types of critical behavior in disordered systems and experi-

mental interest due to the possibility of realizing LR-correlated defects in the orientational glasses,[28] polymers,[29] and disordered solids containing fractal-like defects,[23] dislocations near the sample surface[30] or ^{4}He in porous media such as silica aerogels and xerogels.[31] For this purpose, in this work we performed a field-theoretical description of the effect of LR-correlated quenched defects on the anomalous critical ultrasound attenuation in 3D systems with allowance for both the fluctuation and relaxation attenuation mechanisms without using the ε, δ expansion. We considered in this paper only Ising-like systems that gives to compare the effect of LR-correlated defects with point-like uncorrelated defects on the critical ultrasound attenuation.

In Sec. II, we describe the model of compressible Ising-like systems with LR-correlated disorder and set of dynamical equations which are used for description of the critical sound propagation in solids. We also give in Sec. II the results of RG expansions in a two-loop approximation for ultrasound scaling functions, and β and γ functions of the Callan-Symanzik RG equations. The FP's corresponding to the stability of various types of critical behavior and the critical exponents are determined in Sec. III. Section IV is devoted to analysis of the ultrasound critical characteristics and discussion of the main results.

II. MODEL AND DESCRIPTION OF DYNAMICAL EQUATIONS RG EXPANSIONS

For description of the critical sound propagation in solids the model of compressible media is used. Interaction of the order parameter with elastic deformations plays a significant role in the critical behavior of the compressible system. It was shown for the first time in Ref. 32 that the critical behavior of a system with elastic degrees of freedom is unstable with respect to the connection of the order parameter with acoustic modes and a first-order phase transition is realized. However, the conclusions of Ref. 32 are only valid at low pressures. It was shown in Ref. 33 that in the range of high pressures, beginning from a threshold value of pressure, the deformational effects induced by the external pressure lead to a change in type of the phase transition.

The Hamiltonian of a disordered compressible Ising model can be written as

$$H = H_{el} + H_{op} + H_{int} + H_{imp}, \quad (1)$$

consisting of four contributions.

The contribution H_{el} in (1) of the deformation degrees of freedom is determined as

$$H_{el} = \frac{1}{2} \int d^d x \left(C_{11}^0 \sum_\alpha u_{\alpha\alpha}^2 + 2C_{12}^0 \sum_{\alpha\beta} u_{\alpha\alpha} u_{\beta\beta} + 4C_{44}^0 \sum_{\alpha<\beta} u_{\alpha\beta}^2 \right), \quad (2)$$

where $u_{\alpha\beta}(x)$ are components of the strain tensor and C_{ij}^k are the elastic moduli. The use of an isotropy approximation for H_{el} is caused by the fact that in the critical region the behavior of Hamiltonian parameters of anisotropic system is determined by an isotropic FP of RG transformations, while the anisotropy effects are negligible.[34] This phenomenon of the

increase in system symmetry in the critical point is well known as asymptotic symmetry.

H_{op} is a magnetic part in the appropriate Ginzburg–Landau-Wilson form:

$$H_{op} = \int d^d x \left[\frac{1}{2} \tau_0 S^2 + \frac{1}{2} (\nabla S)^2 + \frac{1}{4} u_0 S^4 \right], \qquad (3)$$

where $S(x)$ is the Ising-field variable which is associated with the spin order parameter, u_0 is a positive constant and $\tau_0 \sim (T - T_{0c})/T_{0c}$ with the mean-field phase transition temperature T_{0c}.

The term H_{int} describes the spin-elastic interaction

$$H_{int} = \int d^d x \left[g_0 \sum_\alpha u_{\alpha\alpha} S^2 \right], \qquad (4)$$

which is bilinear in the spin order parameter and linear in deformations. g_0 is the bare coupling constant.

The term H_{imp} of the Hamiltonian determines the influence of disorder and it is considered in the following form:

$$H_{imp} = \int d^d x [\Delta\tau(x) S^2] + \int d^d x \left[h(x) \sum_\alpha u_{\alpha\alpha} \right], \qquad (5)$$

where the random Gaussian variables $\Delta\tau(x)$ and $h(x)$ determine the local transition temperature fluctuations and induced random stress, respectively. The second moment of distribution of the $\Delta\tau(x)$ fluctuations $g(\boldsymbol{x}-\boldsymbol{y}) = \langle\langle \Delta\tau(\boldsymbol{x})\Delta\tau(\boldsymbol{y}) \rangle\rangle \sim |\boldsymbol{x}-\boldsymbol{y}|^{-a}$ characterizes the LR correlation effects realizing in defects with complex structure. The second moment of distribution of the $h(x)$ fluctuations $C(\boldsymbol{x}-\boldsymbol{y}) = \langle\langle h(\boldsymbol{x})h(\boldsymbol{y}) \rangle\rangle$ presets a renormalization of the elastic moduli and the coupling constant in the spin-elastic interaction.

To perform calculations, it is convenient to use the Fourier components of the deformation variables in the form:

$$u_{\alpha\beta} = u_{\alpha\beta}^{(0)} + V^{-1/2} \sum_{q \neq 0} u_{\alpha\beta}(q) \exp(iqx), \qquad (6)$$

where q is the wavevector, V is the volume, $u_{\alpha\beta}^{(0)}$ is an uniform part of the deformation tensor, and $u_{\alpha\beta}(q) = i/2[q_\alpha u_\beta + q_\beta u_\alpha]$. Then the normal-mode expansion is introduced as

$$\mathbf{u}(q) = \sum_\lambda \mathbf{e}_\lambda(q) Q_{q,\lambda},$$

with the normal coordinate $Q_{q,\lambda}$ and polarization vector $\mathbf{e}_\lambda(q)$. We carry out the integration in the partition function with respect to the nondiagonal components of the uniform part of the deformation tensor $u_{\alpha\beta}^{(0)}$, which are insignificant for the critical behavior of the system in an elastically isotropic medium. After all of the transformations, the effective Hamiltonian of the system has become in the form of a functional for the spin order parameter $S(q)$ and the normal coordinates $Q_\lambda(q)$:

$$\widetilde{H} = \frac{1}{2} \int d^d q (\tau_0 + q^2) S_q S_{-q} + \int d^d q\, h_q Q_{-q,\lambda}$$

$$+ a_0 \int d^d q q^2 Q_{q,\lambda} Q_{-q,\lambda} + \frac{1}{2} \int d^d q \Delta\tau_q S_{q_1} S_{q_1-q}$$

$$+ \frac{1}{4} u_0 \int d^d q S_{q_1} S_{q_2} S_{q_3} S_{-q_1-q_2-q_3} - \frac{1}{2} \mu_0 \int d^d q (S_q S_{-q})$$

$$\times (S_q S_{-q}) - g_0 \int d^d q q Q_{-q,\lambda} S_{q_1} S_{q-q_1}, \qquad (7)$$

where

$$\mu_0 = \frac{3 g_0^2}{V(4 C_{12}^0 - C_{11}^0)}, \quad a_0 = \frac{C_{11}^0 + 4 C_{12}^0 - 4 C_{44}}{4V}.$$

The Fourier transformation of the correlation function for distribution defects in system $g(\mathbf{x}) \sim |\mathbf{x}|^{-a}$ gives $g(\mathbf{k}) = v_0 + w_0 k^{a-d}$ for small k. $g(\mathbf{k})$ must be positive definite, therefore, if $a > d$, then the w_0 term is irrelevant, $v_0 \geq 0$, and the Hamiltonian (7) describes the model with SR-correlated defects, while if $a < d$, then the w_0 term is dominant at small k and $w_0 \geq 0$ and characterizes a large influence of LR-correlated defects on the critical behavior.

The critical dynamics of the compressible system in the relaxational regime can be described by the Langevin equations[35] for the spin order parameter S_q and phonon normal coordinates $Q_{q,\lambda}$:

$$\dot{S}_q = -\Gamma_0 \frac{\partial \widetilde{H}}{\partial S_{-q}} + \xi_q + \Gamma_0 h_S,$$

$$\ddot{Q}_{q,\lambda} = -\frac{\partial \widetilde{H}}{\partial Q_{-q,\lambda}} - q^2 D_0 \dot{Q}_{q,\lambda} + \eta_q + h_Q, \qquad (8)$$

where Γ_0 and D_0 are the bare kinetic coefficients, $\xi_q(x,t)$ and $\eta_q(x,t)$ are Gaussian white noises, and h_S and h_Q are the fields thermodynamically conjugated to the spin and deformation variables, respectively.

The quantities of interest are the response functions $G(q,\omega)$ and $D(q,\omega)$ of the spin and deformation variables, respectively. It can be obtained by linearization in correspondent fields that

$$D(q,\omega) = \frac{\delta[\langle Q_{q,\omega,\lambda} \rangle]}{\delta h_Q} = [\langle Q_{q,\omega,\lambda} Q_{-q,-\omega,\lambda} \rangle], \qquad (9)$$

$$G(q,\omega) = \frac{\delta[\langle S_{q,\omega} \rangle]}{\delta h_S} = [\langle S_{q,\omega} S_{-q,-\omega} \rangle], \qquad (10)$$

where $\langle \ldots \rangle$ denotes averaging over Gaussian white noises, $[\ldots]$ denotes averaging over random fields $\Delta\tau_{-q}$ and h_q that are specified by structural defects.

The response functions may be expressed in terms of self-energy parts:

$$G^{-1}(q,\omega) = G_0^{-1}(q,\omega) + \Pi(q,\omega),$$

FIG. 1. Diagrammatic representation of $\Sigma(q,\omega)$ in a two-loop approximation. (a) Solid line corresponds to $G_0(q,\omega)$; (b) lines with a cross, to $C_0(q,\omega) = 2\Gamma_0^{-1}[(\omega/\Gamma_0)^2 + (q^2 + \tau_0)^2]^{-1}$; (c) vertex with dashed line, to $v = [(\Delta\tau)^2]$; (d) vertex with waved line, to w, which characterized long-range correlation of disorder with wave vector q^{a-d}; and (e) line, to relaxation insertion $M^2\delta_{q,0}$.

$$D^{-1}(q,\omega) = D_0^{-1}(q,\omega) + \Sigma(q,\omega), \qquad (11)$$

where the free response functions $G_0(q,\omega)$ and $D_0(q,\omega)$ have the forms

$$D_0(q,\omega,\lambda) = 1/(\omega^2 - aq^2 - i\omega D_0 q^2),$$

$$G_0(q,\omega) = 1/[i\omega/\Gamma_0 + (\tau_0 + q^2)].$$

In the low-temperature phase, the response functions contain an additional relaxation contribution caused by the presence in the spin density

$$S_q = M\delta_{q,0} + \varphi_q \qquad (12)$$

of an additional part connected with the magnetization

$$M = \begin{cases} 0, & T > T_c, \\ B|T - T_c|^\beta, & T < T_c, \end{cases} \qquad (13)$$

where B is the phenomenological relaxation parameter and φ_q is the fluctuation part of the order parameter.

The characteristics of the critical sound propagation are defined by means of the response function[36] $D(q,\omega)$. Thus, the coefficient of ultrasonic attenuation is determined through the imaginary part of $\Sigma(q,\omega)$:

$$\alpha(\omega,\tau) \sim \omega \, \text{Im} \, \Sigma(0,\omega), \qquad (14)$$

and the sound velocity dispersion is expressed through its real part,

$$c^2(\omega,\tau) - c^2(0,\tau) \sim \text{Re}[\Sigma(0,\omega) - \Sigma(0,0)]. \qquad (15)$$

We calculated $\Sigma(q,\omega)$ in the two-loop approximation. The diagrammatic representation of $\Sigma(q,\omega)$ is presented in Fig. 1.

The Feynman diagrams involve momentum integrations in dimension d (in our case, $d = 3$). Near the critical point the correlation length ξ increases infinitely. When $\xi^{-1} \ll \Lambda$, where Λ is a cut-off in the momentum-space integrals (the cut-off Λ serves to specify the basic length scale), the vertex functions are expected to display an asymptotic scaling behavior for wave numbers $q \ll \Lambda$. Therefore, one is lead to consider the vertex functions in the limit $\Lambda \to \infty$. The use of the RG scheme removes all divergences which arise in thermodynamic variables and kinetic coefficients in this limit.

To calculate the attenuation coefficient (14) and the ultrasound velocity dispersion (15) and to eliminate the diver-

gences in $\Sigma(q,\omega)$ at $q \to 0$, we used the matching method which was introduced for statics in Ref. 37 and then generalized for critical dynamics in Ref. 38. First, we use the dynamical scaling property of the response function

$$D(q,\omega,\tau) = e^{(2-\eta)l}D(qe^l,(\omega/\Gamma_0)e^{zl},\tau e^{l/\nu}), \qquad (16)$$

and then we calculate the right-hand side of this equation for some value $l^* = l$, where the arguments do not all vanish simultaneously.[37] The choice of l^* is determined by the condition

$$[(\omega/\Gamma_0)e^{zl^*}]^{4/z} + [(\tau e^{l^*/\nu})^{2\nu} + q^2 e^{2l^*}]^2 = 1, \qquad (17)$$

which permits to achieve the main purpose of the RG transformations, namely, to find a relation between the behavior of the system in the precritical regime at a low value of the reduced temperature τ and the behavior of the system in a regime far from the critical mode, i.e., without divergences in $\Sigma(q,\omega)$. As it was demonstrated in Ref. 38, the matching condition (17) provides an infrared cut-off for all diverging quantities. The particular form of the matching condition (17) permits an explicit solution for l^* in the form of a functional dependence on ω and τ, which is specified by the dynamic critical exponent z and the static critical exponent ν for the correlation length,

$$e^{l^*} = \tau^{-\nu}[1 + (y/2)^{4/z}]^{-1/4} \equiv \tau^{-\nu}F(y), \qquad (18)$$

where the abbreviation $y = \omega\tau^{-z\nu}/\Gamma_0$. is introduced as the argument of the $F(y)$ function.

The response function $D[qe^l,(\omega/\Gamma_0)e^{zl},\tau e^{l/\nu}]$ on the right-hand side of (16) is represented by Dyson Eq. (11) and the following scaling relationships for the imaginary and real components of self-energy part are valid:[36]

$$\frac{\text{Im} \, \Sigma(\omega)}{\omega} = e^{l(\alpha + z\nu)/\nu}\frac{\text{Im} \, \Sigma(\omega e^{zl})}{\omega e^{zl}}, \qquad (19)$$

$$\text{Re}[\Sigma(0,\omega) - \Sigma(0,0)] = e^{l\alpha/\nu} \, \text{Re}[\Sigma(0,\omega e^{zl}) - \Sigma(0,0)], \qquad (20)$$

where α is the exponent for heat capacity. It may be argued[38] that condition (17) with the well-known expression for the susceptibility[12] provides an infrared cut-off for all divergent values.

It was shown in later theoretical works[9,39] that in asymptotic regions with $\tau \to 0$ and $\omega \to 0$ the coefficient of attenuation and the sound velocity dispersion are described by a simple scaling functions of the variable y only. The experimental investigations performed on the crystals Gd[8] and MnP[9] confirmed the validity of the concepts of dynamical scaling.

In the asymptotic limit ($\tau \to 0$, $\omega \to 0$), the expression for the imaginary part of $\Sigma(\omega, \tau)$ can be defined by a scaling function $\phi(y)$

$$\text{Im } \Sigma(\omega)/\omega = \tau^{-\alpha - z\nu} \phi(y), \tag{21}$$

and the expression for the real part of $\Sigma(\omega, \tau)$ can be determined using another scaling function $f(y)$

$$\text{Re}[\Sigma(0, \omega) - \Sigma(0, 0)] = \tau^{-\alpha}[f(y) - f(0)]. \tag{22}$$

The substitution of e^{l^*} from Eq. (18) into the right-hand side of the expressions (19) and (20) allows to calculate the $\phi(y)$ and $f(y)$ scaling functions.

The dynamic scaling functions calculated in the two-loop approximation have the forms:

$$
\begin{aligned}
\phi(y) = {} & \frac{g^{*2}\Gamma_0}{\pi} \frac{F^{\alpha/\nu + 1/2\nu - z}}{y^2} \left[1 - \frac{(\Delta + 1)^{1/2}}{\sqrt{2}} \right] \\
& - \frac{3g^{*2}u^*\Gamma_0^2}{\sqrt{2}\pi^2} \frac{F^{\alpha/\nu + 1/\nu - 2z}}{y^3} (\Delta - 1)^{1/2} \\
& \times \left[1 - \frac{(\Delta + 1)^{1/2}}{\sqrt{2}} \right] - \frac{g^{*2}v^*\Gamma_0}{12\pi^3} \frac{F^{\alpha/\nu - z}}{y^2} \ln \Delta \\
& - M^2 \frac{3g^{*2}u^*\Gamma_0}{2\pi} \frac{F^{\alpha/\nu - 1/2\nu - z}}{y^2} \left[1 - \frac{(\Delta + 1)^{1/2}}{\sqrt{2}\Delta} \right] \\
& - \frac{w^*\Gamma_0^2}{4\pi^3} \frac{F^{(\alpha + a + 1)/\nu - z}}{y^2} \frac{\Gamma(a - 1)\Gamma(3/2 - a/2)}{\Gamma(3/2)} \\
& \times \Gamma(1 - a/2) \left\{ 1 - \cos\left[\frac{a - 3}{2} \arctan\left(\frac{yF^{1/\nu - z}}{2} \right) \right] \right\},
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
f(y) = {} & \frac{g^{*2}\Gamma_0^2}{\pi} \frac{F^{\alpha/\nu + 1/2\nu - z}}{y} \left[\frac{(\Delta - 1)^{1/2}}{\sqrt{2}} \right] \\
& - \frac{3g^{*2}u^*\Gamma_0^3}{\pi^2} \frac{F^{\alpha/\nu + 1/\nu - 2z}}{y^2} \left[\frac{(\Delta + 1)^{1/2}}{\sqrt{2}} - 1 \right] \\
& - M^2 \frac{3g^{*2}u^*\Gamma_0^2}{2\pi} \frac{F^{\alpha/\nu - 1/2\nu - z}}{y} \left[\frac{(\Delta - 1)^{1/2}}{\Delta\sqrt{2}} \right] \\
& + \frac{g^{*2}v^*\Gamma_0^2}{12\pi^3} \frac{F^{\alpha/\nu - z}}{y} \arctan(\Delta^2 - 1)^{1/2} \\
& - \frac{w^*\Gamma_0^2}{4\pi^3} \frac{F^{(\alpha + a + 1)/\nu - z}}{y^2} \frac{\Gamma(a - 1)\Gamma(3/2 - a/2)}{\Gamma(3/2)} \\
& \times \Gamma(1 - a/2) \sin\left[\frac{a - 3}{2} \arctan\left(\frac{yF^{1/\nu - z}}{2} \right) \right],
\end{aligned}
$$

$$\Delta = \left[1 + \frac{y^2 F^{2z - 2/\nu}}{4} \right]^{1/2}, \tag{24}$$

where g^*, u^*, v^* and w^* are values of the interaction vertices in the FP of the RG transformations that corresponds to the critical behavior of the disordered compressible Ising model,[40] generalized in this paper to the case with LR-correlated disorder. The terms in Eqs. (23) and (24) that are proportional to M^2 describe the relaxation contribution for the scaling functions of the attenuation coefficient and the sound velocity dispersion.

In accordance with our paper,[40] the values of g^*, u^*, v^*, and w^* can be obtained from the RG considerations of the replicated Hamiltonian for the disordered compressible Ising model in which we averaged over deformation variables. The final form of this Hamiltonian can be presented as

$$
\begin{aligned}
H_R = {} & \frac{1}{2} \int d^d q (\tau_0 + q^2) \sum_{a=1}^{n} S_q^a S_{-q}^a \\
& + \frac{u_0 - 2g_0^2}{4} \sum_{a=1}^{n} \int d^d\{q_i\} S_{q_1}^a S_{q_2}^a S_{q_3}^a S_{-q_1 - q_2 - q_3}^a \\
& + \frac{g_0^2 - \tilde{\mu}_0}{2} \sum_{a=1}^{n} \int d^d\{q_i\} (S_{q_1}^a S_{-q_1}^a)(S_{q_2}^a S_{-q_2}^a) \\
& - \frac{1}{2} \sum_{a,b=1}^{n} \int d^d\{q_i\} g(q_1 + q_2)(S_{q_1}^a S_{q_2}^a)(S_{q_3}^b S_{-q_1 - q_2 - q_3}^b),
\end{aligned}
\tag{25}
$$

where $g(\mathbf{q}_1 + \mathbf{q}_2) = v_0 + w_0|\mathbf{q}_1 + \mathbf{q}_2|^{a-d}$ is the Fourier transformation of the correlation function for the quenched random distribution of defects in system. The Hamiltonian in (25) is a functional of n replications of the original order parameter. The properties of the original disordered system after the RG transformations of H_R are obtained in the replica number limit $n \to 0$.

The effective parameter of interaction $\tilde{u}_0 = u_0 - 2g_0^2$ arising in the Hamiltonian (25) from influence of spin-elastic interaction determined by the parameter g_0 can take both positive and negative values. As a result, the Hamiltonian in (25) describes a phase transitions both second order and first order. The tricritical behavior is realized in the system for $\tilde{u}_0 = 0$. Furthermore, as it was shown in Ref. 40, the dependence of the effective interaction in (25) determined by the vertex $g_0^2 - \tilde{\mu}_0(P)$ on outside pressure P can lead to the phase transition of second order for $P > P_t$ and lead to the phase transition of first order for $P < P_t$, where P_t is a tricritical value of outside pressure. But we restrict ourselves in this paper by consideration of the critical behavior only.

As is known, in the field-theory approach[41] the asymptotic critical behavior of systems in the fluctuation region is determined by the Callan-Symanzik RG equation for the vertex parts of the irreducible Green's functions. To calculate the β functions and the critical exponents as scaling γ functions of the renormalized interaction vertices $\tilde{u}$, $\tilde{g}^2 = g^2 - \tilde{\mu}$, v, and w appearing in the RG equation, we used the method based on

TABLE I. Coefficients for the β functions in Eq. (26).

a	b_1	b_2	b_3	b_4	b_5	b_6	b_7
3.00	1.851852	9.703704	3.425924	6.851852	1.000000	2.000000	2.666664
2.90	1.751381	9.149428	3.047432	6.456936	1.632512	1.836920	2.376632
2.80	1.662830	8.671819	2.747992	6.117964	2.148800	1.696968	2.190416
2.70	1.584520	8.260292	2.508396	5.827228	2.584256	1.576244	2.086520
2.60	1.515077	7.906550	2.313792	5.578740	2.967744	1.471748	2.053896
2.50	1.453357	7.604029	2.151672	5.367788	3.325888	1.381120	2.089720
2.40	1.398383	7.347527	2.010060	5.190684	4.022776	1.302500	2.199488
2.30	1.349314	7.132943	1.875288	5.044576	4.904458	1.234384	2.396896
2.20	1.305402	6.957111	1.728540	4.927324	5.918144	1.175580	2.708976
2.10	1.265968	6.817670	1.539252	4.837412	7.035808	1.125124	3.183336
2.00	1.230378	6.713001	1.250616	4.773916	8.216616	1.082260	3.905832

the Feynman diagram technique and the renormalization procedure.[42]

As a result, we obtained the β and γ functions in the two-loop approximation in the form of the expansion series in renormalized vertices $\tilde{u}$, $\tilde{g}^2$, v, and w. We list here the resulting expansions:

$$\beta_{\tilde{u}}(\tilde{u},v,w) = -\tilde{u} + 9\tilde{u}^2 - 24\tilde{u}v - 16(3f_1 - f_2)\tilde{u}w - \frac{308}{9}\tilde{u}^3$$

$$- \frac{5920}{27}\tilde{u}v^2 + \frac{1664}{9}\tilde{u}^2 v + 16(b_1 + b_2)\tilde{u}^2 w$$

$$- 64b_3\tilde{u}w^2 - 64b_4\tilde{u}vw,$$

$$\beta_{\tilde{g}}(\tilde{u},\tilde{g},v,w) = -\tilde{g}^2 + 2\tilde{g}^4 + 6\tilde{u}\tilde{g}^2 - 8v\tilde{g}^2 - 16f_1 w\tilde{g}^2 - \frac{92}{9}\tilde{u}^2\tilde{g}^2$$

$$+ \frac{224}{9}\tilde{u}v\tilde{g}^2 - \frac{736}{27}v^2\tilde{g}^2 - \frac{736}{27}b_5 w^2\tilde{g}^2$$

$$+ \frac{112}{9}b_6\tilde{u}w\tilde{g}^2 - \frac{736}{27}b_6 vw\tilde{g}^2,$$

$$\beta_v(\tilde{u},v,w) = -v - 16v^2 - 16f_3 w^2 - 16(3f_1 - f_2)vw + 6\tilde{u}v$$

$$- \frac{3040}{27}v^3 - 32b_7 w^3 + \frac{800}{9}\tilde{u}v^2 + 12b_8\tilde{u}w^2$$

$$- 32b_9 vw^2 - \frac{92}{9}\tilde{u}^2 v - 32b_{10}v^2 w + 24b_{11}\tilde{u}vw,$$

$$\beta_w(\tilde{u},v,w) = -(4-a)w - 16(f_1 - f_2)w^2 - 8vw + 6\tilde{u}w$$

$$+ 32b_{12}w^3 - 32b_{13}vw^2 - \frac{736}{27}v^2 w - \frac{92}{9}\tilde{u}^2 w$$

$$+ 12b_{14}\tilde{u}w^2 + \frac{368}{9}\tilde{u}vw,$$

$$\gamma_\phi(\tilde{u},v,w) = 8f_2 w + \frac{8}{9}\tilde{u}^2 + \frac{64}{27}v^2 + 8c_1 w^2 - \frac{32}{9}\tilde{u}v - 6c_2\tilde{u}w$$

$$+ 8c_2 vw,$$

$$\gamma_{\phi^2}(\tilde{u},\tilde{g},v,w) = -3\tilde{u} + 4v - 2\tilde{g}^2 + 8f_1 w + 6\tilde{u}^2 + 16v^2 + 2\tilde{g}^4$$

$$+ 8c_3 w^2 + 8c_4 vw - 6c_4\tilde{u}w - 24\tilde{u}v,$$

$$f_1 = \frac{(a-2)(a-4)}{2\,\sin(\pi a/2)},$$

$$f_2 = \frac{(a-2)(a-3)(a-4)}{48\pi\,\sin[\pi(a/2-1)]},$$

$$f_3 = \frac{(2a-5)(2a-7)}{2\,\sin[\pi(a-3/2)]}, \qquad (26)$$

where the coefficients b_i and c_i for different values of parameter a in the range $2 \leq a \leq 3$ are given in Tables I–III. In the series (26) it was used a standard change in variables $\tilde{u} \rightarrow \tilde{u}/J$, $\tilde{g}^2 \rightarrow \tilde{g}^2/J$, $v \rightarrow v/J$, and $w \rightarrow w/J$, where $J = \int d^d q/(q^2+1)^2$ is the one-loop integral.

We should like note that WH model with $a=3$ corresponds to a system with point-like defects. The case with $a=2$ corresponds to a system of straight lines of impurities or straight dislocation lines of random orientations in a sample. The cases with noninteger a with $2<a<3$ are treated in terms of a fractal dimension of impurities. We think that the features of the critical behavior of disordered systems with $2<a<3$ can be displayed in real highly disordered systems when concentration of defects is sufficiently high that effective long-range correlations can occur because of elastic interaction of defects. For values of correlation parameter a less than 1.5, the unavoidable divergencies appear in diagrams for RG β and γ functions in framework of theoretical-field approach with fixed dimension $d=3$, which is used in this paper.

TABLE II. Coefficients for the β-functions in Eq. (26) (the continuation of Table I).

a	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}
3.00	4.000000	8.851848	9.703704	5.703704	−0.851848	1.703704	3.407408
2.90	3.847968	8.018848	9.149424	5.384984	−0.630048	1.471544	2.943084
2.80	3.777176	7.384568	8.671816	5.107714	−0.442952	1.259744	2.519492
2.70	3.785160	6.909368	8.260288	4.866124	−0.278072	1.063416	2.126828
2.60	3.875560	6.565360	7.906552	4.655620	−0.125208	0.878440	1.756888
2.50	4.059204	6.332944	7.604032	4.472486	0.024984	0.701240	1.402484
2.40	4.356584	6.198408	7.347528	4.313686	0.182264	0.528528	1.057056
2.30	4.802876	6.152008	7.132944	4.176734	0.358480	0.357160	0.714324
2.20	5.457744	6.185952	6.957112	4.059572	0.570112	0.184008	0.368020
2.10	6.425424	6.291216	6.817672	3.960514	0.842960	0.005800	0.011596
2.00	7.899532	6.451000	6.713000	3.878172	1.222400	−0.181032	−0.362064

III. FP'S AND VARIOUS TYPES OF CRITICAL BEHAVIOR CRITICAL EXPONENTS

The nature of the critical behavior is determined by the existence of a stable FP satisfying the system of equations

$$\beta_i(\widetilde{u}^*,\widetilde{g}^*,v^*,w^*)=0 \quad (i=1,2,3,4). \qquad (27)$$

It is well known that perturbation series are asymptotic series, and that the vertices describing the interaction of the order parameter fluctuations in the fluctuating region $\tau\rightarrow0$ are large enough so that expressions (26) cannot be used directly. For this reason, to extract the required physical information from the obtained expressions, we employed the Padé-Borel approximation of the summation of asymptotic series extended to the multiparameter case.[16] The direct and the inverse Borel transformations for the multiparameter case have the form

$$f(u_1,u_2,u_3,u_4)=\sum_{i,j,l,k} c_{ijlk}u_1^i u_2^j u_3^l u_4^k$$
$$=\int_0^\infty e^{-t}B(u_1t,u_2t,u_3t,u_4t)dt,$$

TABLE III. Coefficients for the γ-functions in Eqs. (26).

a	c_1	c_2	c_3	c_4
3.00	0.296288	0.592592	2.080000	4.000000
2.90	0.423680	0.730760	1.683744	3.673840
2.80	0.539712	0.874448	1.425600	3.393936
2.70	0.651712	1.025656	1.207872	3.152488
2.60	0.765696	1.186608	1.016128	2.943496
2.50	0.887040	1.359760	0.837056	2.762240
2.40	1.021088	1.547944	0.656544	2.605000
2.30	1.173728	1.754448	0.456736	2.468768
2.20	1.351808	1.983136	0.211584	2.351160
2.10	1.563808	2.238656	−0.122112	2.250248
2.00	1.820576	2.526584	−0.624224	2.164520

$$B(u_1,u_2,u_3,u_4)=\sum_{i,j,l,k}\frac{c_{ijlk}}{(i+j+l+k)!}u_1^i u_2^j u_3^l u_4^k. \qquad (28)$$

A series in the auxiliary variable θ is introduced for analytical continuation of the Borel transform of the function:

$$\widetilde{B}(u_1,u_2,u_3,u_4,\theta)=\sum_{k=0}^\infty \theta^k \sum_{i=0}^{k}\sum_{j=0}^{k-i}\sum_{l=0}^{k-i-j}\frac{c_{i,j,l,k-i-j-l}}{k!}u_1^i u_2^j u_3^l u_4^{k-i-j-l},$$
$$(29)$$

to which the [L/M] Padé approximation is applied at the point $\theta=1$. To perform the analytical continuation, the Padé approximant of [L/1] type may be used which is known to provide rather good results for various Landau-Wilson models (see, e.g., Refs. 43 and 44). The property of preserving the symmetry of a system during application of the Padé approximation by the θ method, as in Ref. 43, has become important for multivertices models. We used the [2/1] approximant to calculate the β functions in the two-loop approximation.

However, analysis of the series coefficients for the β_w function has shown that the summation of this series is fairly poor, which resulted in the absence of FP's with $w^*\neq0$ for $a<2.93$. Dorogovtsev[24] found the symmetry of the scaling function for the WH model in relation to the transformation $(u,v,w)\rightarrow(u,v,v+w)$, which gives the possibility of investigating the problem of the existence of FP's with $w^*\neq0$ in the variables $(u,v,v+w)$. In this case, our investigations carried out in Ref. 16 have shown the existence of FP's with $w^*\neq0$ in the whole region where the parameter a changes.

We have found two classes of FP's with $g^*=0$ for an incompressible Ising model and with $g^*>0$ for the compressible Ising model. If a possibility of realization of multicritical behavior in system for $\widetilde{u}^*=u^*-2g^{*2}=0$ or for $\widetilde{g}^{*2}=g^{*2}-\widetilde{u}^*=0$ does not consider in this paper (for critical behavior[40] $\widetilde{u}^*=0$ and $\widetilde{g}^*=g^*$), then for either of the two classes there are three types of FP's in the physical region of parameter space, $\widetilde{u}^*,g^{*2},v^*$, and $v^*+w^*>0$ for different values of a. Type I corresponds to the FP of a pure system

TABLE IV. Stable fixed points and critical exponents for the compressible Ising model with LR-correlated disorder.

a	$\widetilde{u}^*$	$\widetilde{g}^{*2}$	v^*	v^*+w^*	ν	α	β	z
3.01	0.26482	0.02043	0.03448	0.03448	0.6896	−0.0687	0.3561	2.1712
3.00	0.26482	0.02043	0.01393	0.03448	0.6896	−0.0687	0.3561	2.1712
2.90	0.28867	0.06021	0.01993	0.04257	0.7387	−0.2160	0.3806	2.2120
2.80	0.30881	0.05443	0.02510	0.04894	0.7411	−0.2232	0.3805	2.2486
2.70	0.32670	0.04708	0.02968	0.05422	0.7402	−0.2205	0.3785	2.2837
2.60	0.34294	0.03847	0.03380	0.05870	0.7369	−0.2106	0.3749	2.3184
2.50	0.35776	0.02899	0.03752	0.06248	0.7320	−0.1958	0.3703	2.3532
2.40	0.37120	0.02281	0.04086	0.06562	0.7297	−0.1890	0.3668	2.3879
2.30	0.38313	0.01772	0.04380	0.06811	0.7278	−0.1833	0.3634	2.4215
2.20	0.39322	0.01281	0.04631	0.06989	0.7254	−0.1761	0.3597	2.4524
2.10	0.40090	0.00762	0.04829	0.07084	0.7219	−0.1657	0.3557	2.4780
2.00	0.40521	0.00186	0.04959	0.07074	0.7169	−0.1507	0.3511	2.4949

($\widetilde{u}^* \neq 0$, v^*, $w^*=0$), type II is a SR-disorder FP ($\widetilde{u}^*$, $v^* \neq 0$, $w^*=0$), and type III corresponds to LR-disorder FP ($\widetilde{u}^*$, v^*, $w^* \neq 0$).

The type of critical behavior of this disordered system for each value of a is determined by the stability of the corresponding FP. The stability properties of the FP's are controlled by the eigenvalues λ_i of the matrix

$$\Omega_{ij} = \frac{\partial \beta_i}{\partial u_j} \qquad (30)$$

computed at the given FP: a FP is stable if all eigenvalues λ_i are positive. If some of the eigenvalues λ_i are complex numbers, then the real parts of these eigenvalues must be positive for stable FP.

Our calculations showed that FP's which belong to the class of incompressible Ising model with $g^*=0$ are unstable with respect to the influence of elastic deformations. For class of FP's corresponding to the compressible Ising model with $g^* \neq 0$ the values of the stable FP's obtained for $2 \leq a \leq 3$ are presented in Table IV. As one can see from this Table, for the compressible Ising model the LR-disorder FP is stable for values of a in the whole investigated range. The additional calculations for $3 < a < 4$ have shown that only SR-disorder FP is stable in this range. For $a=3$ FP values for vertices $\widetilde{u}$ and $g(k)$ are equal, $\widetilde{u}^*=0.26482$ and $v^*+w^* =0.03448$, and correspond to the SR-disorder FP for compressible Ising model, although $w^* \neq 0$. Similarly, for $a=3$ the LR disorder is marginal, and the critical behavior of the compressible Ising model with LR-correlated disorder, as that of the SR-disordered compressible Ising model, is characterized by the same critical exponents (Table IV).

We have calculated the static critical exponents η and ν for the compressible Ising model with LR-correlated disorder (Table IV), received from the resummed by the generalized Padé-Borel method γ functions in the corresponding stable FP's:

$$\eta = \gamma_\phi(\widetilde{u}^*, \widetilde{g}^*, v^*, w^*),$$

$$\nu = [2 + \gamma_{\phi^2}(\widetilde{u}^*, \widetilde{g}^*, v^*, w^*) - \gamma_\phi(\widetilde{u}^*, \widetilde{g}^*, v^*, w^*)]^{-1}. \qquad (31)$$

Values of the specific-heat exponent α and the exponent β for magnetization which are required for calculation of ultrasound characteristics were determined from scaling relations $\alpha = 2 - d\nu$ and $\beta = (d-2+\eta)\nu/2$ for $2 \leq a \leq 3$ and presented in Table IV. The values of the dynamic critical exponent z also presented in Table IV for $2 \leq a \leq 3$ were taken from our paper,[16] where the critical dynamics of 3D disordered Ising model with LR-correlated disorder was considered for purely relaxational model A. It is caused by fact[45] that the coupling of the order parameter with elastic deformations is irrelevant for the relaxational critical properties of the order parameter in disordered compressible Ising model with a negative specific-heat exponent α.

IV. ANALYSIS OF RESULTS FOR ULTRASOUND CRITICAL CHARACTERISTICS AND CONCLUSIONS

In this work, we employed the Padé-Borel approximation of the summation of asymptotic series for calculation of the ultrasound scaling functions $\phi(y)$ in (24) using approximant [1/1]. The short series for scaling functions $\phi(y)$ were summed on the values of vertices u, v, and w in LR-disorder FP for $2 \leq a \leq 3$ with variable y which was changed in the interval from 10^{-3} to 10^3 by steps with $\Delta y = 0.1$.

The resultant behavior of the dynamic scaling functions $\phi(y)$ and $f(y)$ for individual values of a is shown in Figs. 2(a) and 2(b) on a log-log scale. Depending on the interval of changing variable y, the following asymptotic regions can be distinguished in the behavior of $\phi(y)$: a hydrodynamic region, where $y \sim \omega\xi \sim (q\xi)^z \ll 1$, and a critical region $y \sim \omega\xi \gg 1$, which determines the behavior of the system near the phase transition temperature ($\tau = (T-T_c)/T_c \ll 1$). We have seen from these curves that the correlation properties of a structural disorder does not affect the behavior of the scaling functions $\phi(y)$ in the hydrodynamic region with $y \ll 1$. However, LR-disorder begins to manifest itself in the crossover region $10^{-1} < y < 10$ and it has a drastic effect in the critical region with $y > 10$ ($T \rightarrow T_c$).
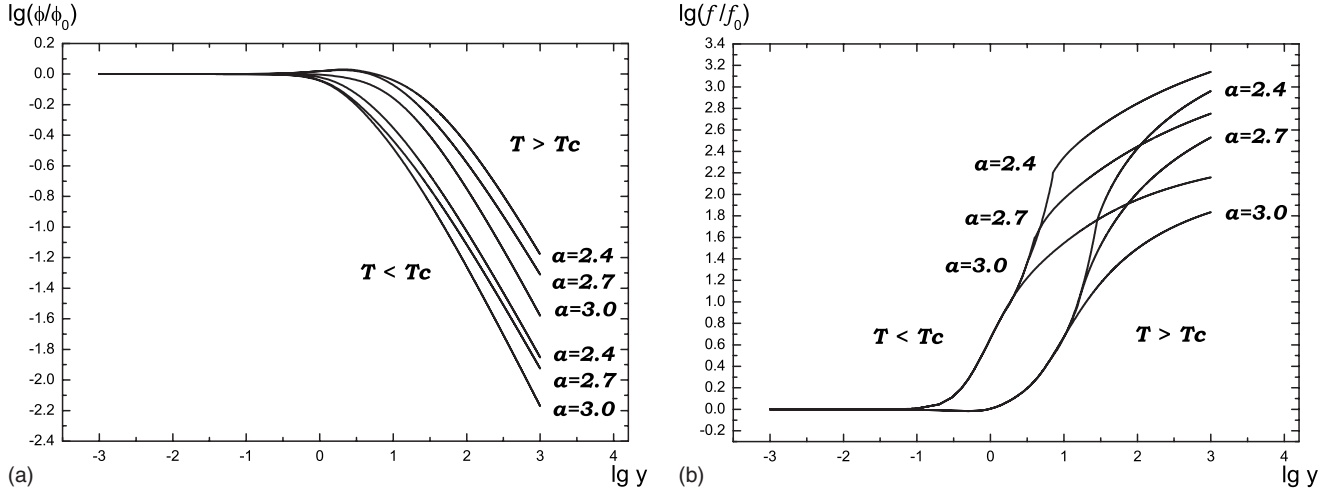
FIG. 2. (a) Scaling functions for the critical sound attenuation $\phi(y)$ and (b) sound velocity dispersion $f(y)$ in a double-logarithmic plot for disordered systems characterized by different values of the correlation parameter a at $T > T_c$ and $T < T_c [\phi_0 = \phi(0), f_0 = f(0)]$.

As follows from Eqs. (14) and (21), the attenuation coefficient can be expressed as

$$\alpha(\omega, \tau) \sim \omega^2 \tau^{-\alpha - \nu z} \phi(y), \qquad (32)$$

and, using Eqs. (15) and (22), we can write the relation for the sound velocity dispersion in the form

$$c^2(\omega, \tau) - c^2(0, \tau) \sim \tau^{-\alpha}[f(y) - f(0)]. \qquad (33)$$

The results of the calculation of the asymptotic dependences of the attenuation coefficient and the sound velocity dispersion for the critical and hydrodynamic regions are given in the Table V. The characteristics of their frequency and temperature dependences were determined in the range $10^{-3} \leq y \leq 10^{-1}$ for the hydrodynamic regime and in the range $10 \leq y \leq 10^3$ for the critical regime. Note that, according to,[38] the real temperature range $10^{-3} \leq \tau \leq 10^{-1}$ in ultrasonic studies of phase transitions corresponds to the range

$1 \leq y \leq 10^2$, i.e., it covers the crossover region and the beginning of the critical region (precritical regime).

As it follows from the Table V, the increase in correlation effects for disorder characterized by decrease in parameter a values leads to systematical increase in the values of exponents $k_\omega^{(\alpha)(c)}$ and $k_\tau^{(\alpha)(c)}$, which we can introduce for description of the critical anomalies of the frequency and temperature dependences of the attenuation coefficient $\alpha \sim \omega^{k_\omega^{(\alpha)}} \tau^{-k_\tau^{(\alpha)}}$ and the sound velocity dispersion $c^2(\omega, \tau) - c^2(0, \tau) \sim \omega^{k_\omega^{(c)}} \tau^{-k_\tau^{(c)}}$. So, for the attenuation coefficient the values of exponents $k_\omega^{(\alpha)} = 1.21$ and $k_\tau^{(\alpha)} = 0.24$ for $T > T_c$ and $k_\omega^{(\alpha)} = 1.11$ and $k_\tau^{(\alpha)} = 0.10$ for $T < T_c$ in the case of point-like uncorrelated defects characterized by parameter $a \geq 3.0$, $k_\omega^{(\alpha)} = 1.29$ and $k_\tau^{(\alpha)} = 0.29$ for $T > T_c$ and $k_\omega^{(\alpha)} = 1.21$ and $k_\tau^{(\alpha)} = 0.13$ for $T < T_c$ in the case of complex structure defects characterized, for example, by value of the parameter $a = 2.7$, and $k_\omega^{(\alpha)} = 1.30$ and $k_\tau^{(\alpha)} = 0.32$ for $T > T_c$ and $k_\omega^{(\alpha)} = 1.21$

TABLE V. Asymptotic behavior of the sound attenuation coefficient $\alpha(\omega, \tau)$ in the critical, precritical, and hydrodynamic regimes for pure and disordered systems

	$T < T_c$			$T > T_c$		
System	Hydrodynamic	Precritical	Critical	Hydrodynamic	Precritical	Critical
Pure	$\omega^{2.00} \tau^{-1.38}$	$\omega^{1.08} \tau^{-0.21}$	$\omega^{0.98} \tau^{-0.08}$	$\omega^{2.00} \tau^{-1.38}$	$\omega^{1.20} \tau^{-0.37}$	$\omega^{1.05} \tau^{-0.17}$
$a = 3.0$	$\omega^{2.00} \tau^{-1.43}$	$\omega^{1.21} \tau^{-0.24}$	$\omega^{1.11} \tau^{-0.10}$	$\omega^{2.00} \tau^{-1.43}$	$\omega^{1.37} \tau^{-0.49}$	$\omega^{1.21} \tau^{-0.24}$
$a = 2.9$	$\omega^{2.00} \tau^{-1.43}$	$\omega^{1.30} \tau^{-0.27}$	$\omega^{1.21} \tau^{-0.12}$	$\omega^{2.00} \tau^{-1.43}$	$\omega^{1.46} \tau^{-0.54}$	$\omega^{1.29} \tau^{-0.28}$
$a = 2.8$	$\omega^{2.00} \tau^{-1.44}$	$\omega^{1.30} \tau^{-0.28}$	$\omega^{1.21} \tau^{-0.13}$	$\omega^{2.00} \tau^{-1.44}$	$\omega^{1.46} \tau^{-0.55}$	$\omega^{1.29} \tau^{-0.29}$
$a = 2.7$	$\omega^{2.00} \tau^{-1.47}$	$\omega^{1.30} \tau^{-0.29}$	$\omega^{1.21} \tau^{-0.13}$	$\omega^{2.00} \tau^{-1.47}$	$\omega^{1.46} \tau^{-0.56}$	$\omega^{1.29} \tau^{-0.29}$
$a = 2.6$	$\omega^{2.00} \tau^{-1.50}$	$\omega^{1.30} \tau^{-0.30}$	$\omega^{1.21} \tau^{-0.14}$	$\omega^{2.00} \tau^{-1.50}$	$\omega^{1.46} \tau^{-0.57}$	$\omega^{1.30} \tau^{-0.30}$
$a = 2.5$	$\omega^{2.00} \tau^{-1.53}$	$\omega^{1.30} \tau^{-0.32}$	$\omega^{1.21} \tau^{-0.15}$	$\omega^{2.00} \tau^{-1.53}$	$\omega^{1.46} \tau^{-0.59}$	$\omega^{1.30} \tau^{-0.30}$
$a = 2.4$	$\omega^{2.00} \tau^{-1.55}$	$\omega^{1.30} \tau^{-0.34}$	$\omega^{1.21} \tau^{-0.16}$	$\omega^{2.00} \tau^{-1.55}$	$\omega^{1.47} \tau^{-0.63}$	$\omega^{1.30} \tau^{-0.32}$
$a = 2.3$	$\omega^{2.00} \tau^{-1.58}$	$\omega^{1.32} \tau^{-0.37}$	$\omega^{1.21} \tau^{-0.17}$	$\omega^{2.00} \tau^{-1.58}$	$\omega^{1.51} \tau^{-0.71}$	$\omega^{1.30} \tau^{-0.34}$
$a = 2.2$	$\omega^{2.00} \tau^{-1.60}$	$\omega^{1.32} \tau^{-0.39}$	$\omega^{1.22} \tau^{-0.19}$	$\omega^{2.00} \tau^{-1.60}$	$\omega^{1.53} \tau^{-0.76}$	$\omega^{1.31} \tau^{-0.37}$
$a = 2.1$	$\omega^{2.00} \tau^{-1.62}$	$\omega^{1.33} \tau^{-0.42}$	$\omega^{1.23} \tau^{-0.21}$	$\omega^{2.00} \tau^{-1.62}$	$\omega^{1.56} \tau^{-0.83}$	$\omega^{1.32} \tau^{-0.41}$
$a = 2.0$	$\omega^{2.00} \tau^{-1.64}$	$\omega^{1.34} \tau^{-0.45}$	$\omega^{1.24} \tau^{-0.23}$	$\omega^{2.00} \tau^{-1.64}$	$\omega^{1.60} \tau^{-0.92}$	$\omega^{1.33} \tau^{-0.44}$

TABLE VI. Asymptotic behavior of the sound velocity dispersion $c^2(\omega,\tau)-c^2(0,\tau)$ in the critical, precritical, and hydrodynamic regimes for pure and disordered systems

System	$T < T_c$			$T > T_c$		
	Hydrodynamic	Precritical	Critical	Hydrodynamic	Precritical	Critical
Pure	$\omega^{2.00}\tau^{-2.65}$	$\omega^{0.30}\tau^{-0.49}$	$\omega^{0.11}\tau^{-0.25}$	$\omega^{2.00}\tau^{-2.65}$	$\omega^{1.08}\tau^{-1.48}$	$\omega^{0.34}\tau^{-0.54}$
$a=3.0$	$\omega^{2.00}\tau^{-2.93}$	$\omega^{0.38}\tau^{-0.51}$	$\omega^{0.25}\tau^{-0.30}$	$\omega^{2.00}\tau^{-2.93}$	$\omega^{1.17}\tau^{-1.68}$	$\omega^{0.46}\tau^{-0.62}$
$a=2.9$	$\omega^{2.00}\tau^{-3.05}$	$\omega^{0.46}\tau^{-0.54}$	$\omega^{0.32}\tau^{-0.33}$	$\omega^{2.00}\tau^{-3.05}$	$\omega^{1.22}\tau^{-1.77}$	$\omega^{0.57}\tau^{-0.72}$
$a=2.8$	$\omega^{2.00}\tau^{-3.11}$	$\omega^{0.46}\tau^{-0.55}$	$\omega^{0.33}\tau^{-0.34}$	$\omega^{2.00}\tau^{-3.11}$	$\omega^{1.32}\tau^{-1.98}$	$\omega^{0.61}\tau^{-0.80}$
$a=2.7$	$\omega^{2.00}\tau^{-3.16}$	$\omega^{0.46}\tau^{-0.56}$	$\omega^{0.33}\tau^{-0.35}$	$\omega^{2.00}\tau^{-3.16}$	$\omega^{1.42}\tau^{-2.18}$	$\omega^{0.65}\tau^{-0.88}$
$a=2.6$	$\omega^{2.00}\tau^{-3.21}$	$\omega^{0.46}\tau^{-0.58}$	$\omega^{0.33}\tau^{-0.36}$	$\omega^{2.00}\tau^{-3.21}$	$\omega^{1.49}\tau^{-2.34}$	$\omega^{0.66}\tau^{-0.92}$
$a=2.5$	$\omega^{2.00}\tau^{-3.25}$	$\omega^{0.46}\tau^{-0.60}$	$\omega^{0.33}\tau^{-0.37}$	$\omega^{2.00}\tau^{-3.25}$	$\omega^{1.58}\tau^{-2.53}$	$\omega^{0.66}\tau^{-0.94}$
$a=2.4$	$\omega^{2.00}\tau^{-3.30}$	$\omega^{0.46}\tau^{-0.62}$	$\omega^{0.33}\tau^{-0.38}$	$\omega^{2.00}\tau^{-3.30}$	$\omega^{1.62}\tau^{-2.64}$	$\omega^{0.67}\tau^{-0.97}$
$a=2.3$	$\omega^{2.00}\tau^{-3.34}$	$\omega^{0.47}\tau^{-0.65}$	$\omega^{0.33}\tau^{-0.39}$	$\omega^{2.00}\tau^{-3.34}$	$\omega^{1.66}\tau^{-2.75}$	$\omega^{0.67}\tau^{-1.00}$
$a=2.2$	$\omega^{2.00}\tau^{-3.38}$	$\omega^{0.50}\tau^{-0.70}$	$\omega^{0.34}\tau^{-0.39}$	$\omega^{2.00}\tau^{-3.38}$	$\omega^{1.72}\tau^{-2.89}$	$\omega^{0.68}\tau^{-1.03}$
$a=2.1$	$\omega^{2.00}\tau^{-3.41}$	$\omega^{0.51}\tau^{-0.74}$	$\omega^{0.35}\tau^{-0.41}$	$\omega^{2.00}\tau^{-3.41}$	$\omega^{1.76}\tau^{-2.98}$	$\omega^{0.70}\tau^{-1.09}$
$a=2.0$	$\omega^{2.00}\tau^{-3.43}$	$\omega^{0.57}\tau^{-0.87}$	$\omega^{0.37}\tau^{-0.51}$	$\omega^{2.00}\tau^{-3.43}$	$\omega^{1.80}\tau^{-3.06}$	$\omega^{0.75}\tau^{-1.18}$

and $k_\tau^{(\alpha)}=0.16$ for $T < T_c$ in the case of extended linear defects characterized by value of the parameter $a=2.4$. The comparison of these exponent values with the values of $k_\omega^{(\alpha)}=1.05$ and $k_\tau^{(\alpha)}=0.17$ for $T > T_c$ and $k_\omega^{(\alpha)}=0.98$ and $k_\tau^{(\alpha)}=0.08$ for $T < T_c$ calculated in our paper[21] for a pure Ising-like systems shows the strong influence of disorder and its correlation effects on the frequency and temperature dependences of the attenuation coefficient in the vicinity of the critical point.

We must note that the increase in the attenuation coefficient with increase in LR correlations for disorder as the critical temperature is approached is expected to be stronger than for systems with uncorrelated disorder or for the pure systems even in the hydrodynamic region. At the same time in the critical region, the systems with LR-correlated disorder should exhibit stronger both frequency and temperature dependences of the attenuation coefficient than those for sys-

tems with uncorrelated disorder or for pure systems.

The similar conclusions we can make in relation to the frequency and temperature dependences of the sound velocity dispersion in the critical range on basis of those values of exponents $k_\omega^{(c)}$ and $k_\tau^{(c)}$ which are given in Table VI and the values of $k_\omega^{(c)}=0.34$ and $k_\tau^{(c)}=0.54$ for $T > T_c$ and $k_\omega^{(c)}=0.11$ and $k_\tau^{(c)}=0.25$ for $T < T_c$ calculated in paper[21] for pure systems.

These conclusions are supported by the model representation of the results of the numerical calculations of the attenuation coefficient (Fig. 3) for systems with LR-correlated disorder for different values of the parameter a performed at $B=0.3$ and $\omega/\Gamma_0=0.0015$. These values were determined in Ref. [21] when we compared the calculated temperature dependence of the attenuation coefficient and the results of experimental studies of pure FeF_2 samples,[1] which demonstrate Ising-like behavior in the critical region.

A particularly important result of our investigation consists in the predicted manifestation of the dynamical effects of structural defects on anomalous ultrasound attenuation and dispersion over a wider temperature range near the critical temperature (already in the hydrodynamic region) in comparison with other experimental methods,[46] which require a narrow temperature range (of about $\tau \simeq 10^{-4}$) to be studied for revealing these effects. Thus, the results obtained in this paper can serve as a reference point for purposeful experimental investigations of the dynamical effects of structural defects and their correlation properties on the critical behavior of solids using acoustic methods via the detection of the peculiarities of structural defects influence on the frequency and temperature dependences of the ultrasound attenuation and dispersion.
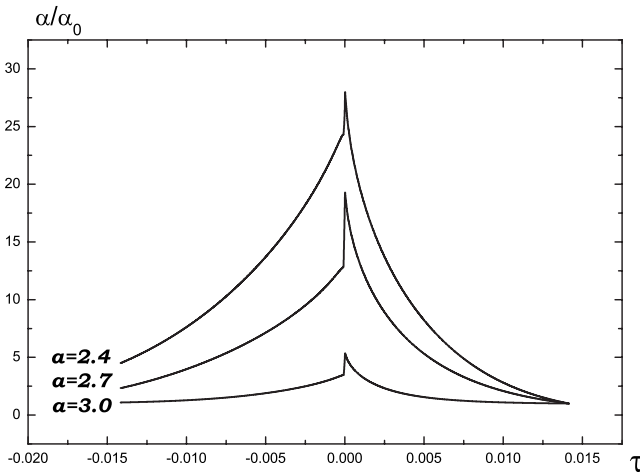

FIG. 3. Temperature dependence of the attenuation coefficient $\alpha(\omega,\tau)$ calculated for systems with LR-correlated disorder characterized by different values of the correlation parameter a at $B=0.3$ and $\omega/\Gamma_0=0.0015$.

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[1] A. Ikushima and R. Feigelson, J. Phys. Chem. Solids **32**, 417 (1971).

[2] Kh. K. Aliev, I. Kh. Kamilov, and A. M. Omarov, Zh. Éksp. Teor. Fiz. **95**, 1896 (1989) [Sov. Phys. JETP **68**, 1096 (1989)].

[3] L. D. Landau and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR **96**, 496 (1954).

[4] A. Pawlak, Phys. Rev. B **44**, 5296 (1991).

[5] A. M. Schorgg and F. Schwabl, Phys. Rev. B **49**, 11682 (1994).

[6] I. K. Kamilov and Kh. K. Aliev, Usp. Fiz. Nauk **168**, 953 (1998) [Phys. Usp. **41**, 865 (1998)].

[7] R. A. Ferrell, B. Mirhashem, and J. K. Bhattacharjee, Phys. Rev. B **35**, 4662 (1987).

[8] T. J. Moran and B. Lüthi, Phys. Rev. B **4**, 122 (1971).

[9] M. Suzuki and T. Komatsubara, J. Phys. C **15**, 4559 (1982).

[10] R. Folk, Yu. Holovatch, and T. Yavors'kii, Phys. Usp. **46**, 169 (2003) [Usp. Fiz. Nauk **173**, 175 (2003)].

[11] A. B. Harris, J. Phys. C **7**, 1671 (1974).

[12] A. Pawlak and B. Fechner, Phys. Rev. B **40**, 9324 (1989).

[13] V. V. Prudnikov, A. V. Ivanov, and A. A. Fedorenko, JETP Lett. **66**, 835 (1997).

[14] V. V. Prudnikov, S. V. Belim, A. V. Ivanov, E. V. Osintsev, and A. A. Fedorenko, Zh. Éksp. Teor. Fiz. **114**, 972 (1998) [Sov. Phys. JETP **87**, 527 (1998)].

[15] V. V. Prudnikov, P. V. Prudnikov, and A. A. Fedorenko, Sov. Phys. JETP **89**, 325 (1999).

[16] V. V. Prudnikov, P. V. Prudnikov, and A. A. Fedorenko, Phys. Rev. B **62**, 8777 (2000).

[17] V. V. Prudnikov, P. V. Prudnikov, and A. A. Fedorenko, Phys. Rev. B **63**, 184201 (2001).

[18] V. V. Prudnikov and P. V. Prudnikov, Sov. Phys. JETP **95**, 550 (2002).

[19] P. V. Prudnikov and V. V. Prudnikov, J. Phys.: Condens. Matter **17**, L485 (2005).

[20] P. V. Prudnikov, V. V. Prudnikov, and E. A. Nosikhin, Phys. Met. Metallogr. **104**, 221 (2007) [Fiz. Met. Metalloved. **104**, 235 (2007)].

[21] P. V. Prudnikov, V. V. Prudnikov, and E. A. Nosikhin, Sov. Phys. JETP **106**, 897 (2008) [Zh. Éksp. Teor. Fiz. **133**, 1027 (2008)].

[22] A. Weinrib and B. I. Halperin, Phys. Rev. B **27**, 413 (1983).

[23] A. L. Korzhenevskii, A. A. Luzhkov, and W. Schirmacher, Phys. Rev. B **50**, 3661 (1994).

[24] S. N. Dorogovtsev, J. Phys. A **17**, L677 (1984).

[25] E. Korutcheva and D. Uzunov, Phys. Status Solidi B **126**, K19 (1984).

[26] E. Korutcheva and F. Javier de la Rubia, Phys. Rev. B **58**, 5153 (1998).

[27] V. V. Prudnikov, P. V. Prudnikov, B. Zheng, S. V. Dorofeev, and Y. Y. Kolesnikov, Prog. Theor. Phys. **117**, 973 (2007).

[28] K. Binder and J. D. Reger, Adv. Phys. **41**, 547 (1992).

[29] V. Blavats'ka, C. von Ferber, and Yu. Holovatch, Phys. Rev. E **64**, 041102 (2001).

[30] M. Altarelli, M. D. Nunez-Regueiro, and M. Papoular, Phys. Rev. Lett. **74**, 3840 (1995).

[31] J. Yoon, D. Sergatskov, J. Ma, N. Mulders, and M. H. W. Chan, Phys. Rev. Lett. **80**, 1461 (1998); M. Chan, N. Mulders, and J. Reppy, Phys. Today **49**(8), 30 (1996); C. Vásquez R., R. Paredes V., A. Hasmy, and R. Jullien, Phys. Rev. Lett. **90**, 170602 (2003).

[32] A. I. Larkin and S. A. Pikin, Zh. Éksp. Teor. Fiz. **56**, 1664 (1969) [Sov. Phys. JETP **29**, 891 (1969)].

[33] Y. Imry, Phys. Rev. Lett. **33**, 1304 (1974).

[34] Yu. A. Izyumov and V. N. Syromyatnikov, *Phase Transitions and Crystal Symmetry* (Nauka, Moscow/Kluwer, Dordrecht, 1990).

[35] G. Meissner, Ferroelectrics **24**, 27 (1980); F. Schwabl and H. Iro, *ibid.* **35**, 27 (1981).

[36] H. Iro and F. Schwabl, Solid State Commun. **46**, 205 (1983).

[37] D. R. Nelson, Phys. Rev. B **14**, 1123 (1976).

[38] R. Folk, H. Iro, and F. Schwabl, Z. Phys. B **27**, 169 (1977).

[39] K. Kawasaki, *Phase Transitions and Critical Phenomena*, (Academic London, 1976), Vol. 5a, p. 165; K. Kawasaki, *Proceedings of Conference on Internal Friction and Ultrasonic Attenuation in Solids* (Tokyo University Press, Tokyo, 1977), p. 29.

[40] V. V. Prudnikov and S. V. Belim, Fiz. Tverd. Tela (St.Petersburg) **43**, 1299 (2001) [Phys. Solid State **43**, 1353 (2001)].

[41] D. Amit, *Field Theory, The Renormalization Group, and Critical Phenomena* (McGraw-Hill, New York, 1978).

[42] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon Press, Oxford, 1996).

[43] S. A. Antonenko and A. I. Sokolov, Phys. Rev. B **49**, 15901 (1994); A. I. Sokolov, K. B. Varnashev, and A. I. Mudrov, Int. J. Mod. Phys. B **12**, 1365 (1998).

[44] G. A. Baker, B. G. Nickel, and D. I. Meiron, Phys. Rev. B **17**, 1365 (1978).

[45] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[46] N. Rosov, C. Hohenemser, and M. Eibschutz, Phys. Rev. B **46**, 3452 (1992).